



Achim, AM., Canagarajah, CN., & Bull, DR. (2005). Complex wavelet domain image fusion based on fractional lower order moments. In *IEEE 8th International Conference on Information Fusion, Philadelphia, USA* (Vol. 1, pp. 515 - 521). Institute of Electrical and Electronics Engineers (IEEE).
<https://doi.org/10.1109/ICIF.2005.1591898>

Peer reviewed version

Link to published version (if available):
[10.1109/ICIF.2005.1591898](https://doi.org/10.1109/ICIF.2005.1591898)

[Link to publication record in Explore Bristol Research](#)
PDF-document

University of Bristol - Explore Bristol Research

General rights

This document is made available in accordance with publisher policies. Please cite only the published version using the reference above. Full terms of use are available:
<http://www.bristol.ac.uk/red/research-policy/pure/user-guides/ebr-terms/>

Complex Wavelet Domain Image Fusion Based on Fractional Lower Order Moments

A.M. Achim

C.N. Canagarajah

D.R. Bull

Department of Electrical & Electronic Engineering
University of Bristol, BS8 1UB, Bristol, UK
Alin.Achim@bristol.ac.uk

Abstract – We describe an image fusion algorithm for data exhibiting heavy tails with no convergent second- or higher-order moments. Our developments rely on recent results showing that wavelet decomposition coefficients of images are best modeled by alpha-stable distributions, a family of heavy-tailed densities. Thus, in the multiscale wavelet domain we develop a novel fusion rule based on fractional lower order moments (FLOM's). Simulation results show that our method achieves better performance in comparison with previously proposed pixel-level fusion approaches.

Keywords: Image fusion, wavelet decomposition, alpha-stable distributions, parameter estimation, symmetric covariation, fractional lower order moments.

1 Introduction

The purpose of image fusion is to combine information from multiple images of the same scene into a single image that ideally contains all the important features from each of the original images. The resulting fused image will be thus more suitable for human and machine perception or for further image processing tasks. Many image fusion schemes have been developed in the past. In general, these schemes can be roughly classified into pixel-based and region-based methods. In [1] it has been shown that comparable results can be achieved using both types of methods with added advantages for the region based approaches, mostly in terms of the possibility of implementing more intelligent fusion rules. On the other hand, pixel based algorithm are simpler and thus easier to implement.

Recently, there has been considerably interest in using the wavelet transform as a powerful framework for implementing image fusion algorithms [1, 2, 3, 4]. Specifically, methods based on multiscale decompositions consist of three main steps: First, the set of im-

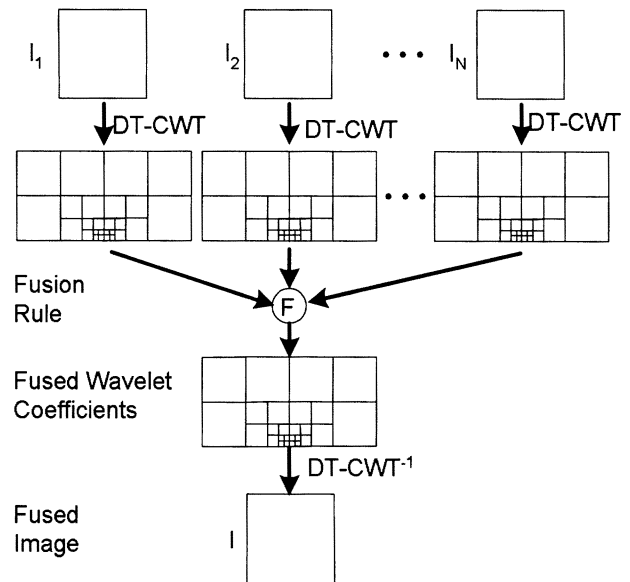


Figure 1: Pixel-based image fusion scheme using the DTDWT.

ages to be fused are analyzed by means of the wavelet transform, then the resulting wavelet coefficients are fused through an appropriately designed rule, and finally, the fused image is synthesized from the processed wavelet coefficients through the inverse wavelet transform. This process is depicted in Fig. 1.

In this context, after more than ten years since its inception as an image fusion algorithm, the method proposed by Burt and Kolczynsky [2] remains one of the most effective, yet simple and easy to implement. Their method essentially consists in calculating a normalised correlation (match measure) between the two images' subbands over a small local area. Then, the fused coefficient is calculated from this measure and the local variance (saliency measure) via a weighted average of the two images' coefficients.

In this work, we propose a generalization of the above method for the case when the data to be fused exhibit heavy tails with no convergent second- or higher-order moments. Specifically, our developments are based on recently published results showing that wavelet decomposition coefficients of images are best modelled by symmetric alpha-stable $S\alpha S$ distributions, a family of heavy-tailed densities [5, 6]. Unfortunately, only moments of orders less than α ($0 < \alpha < 2$) can be defined for the general alpha-stable family members. Consequently, as measure of salience we employ the dispersion of the alpha-stable distribution computed in a neighbourhood around the reference coefficients. Also, due to the lack of finite variance, covariances do not exist either on the space of $S\alpha S$ random variables. Instead, quantities like covariations or codifferences, which under certain circumstances play analogous roles for $S\alpha S$ random variables to the one played by covariance for Gaussian random variables have been introduced. Therefore, we propose the use of symmetrized and normalised versions of these quantities, which enable us to define a new match measure for $S\alpha S$ random vectors. In our implementation we make use of the dual-tree complex wavelet transform (DTCWT) that has been shown to offer near shift invariance and improved directional selectivity compared to the standard wavelet transform [7, 8]. Due to these properties, image fusion methods implemented in the complex wavelet domain have been shown to outperform those implemented using the discrete wavelet transform [4].

The paper is organized as follows: In Section 2, we provide some necessary preliminaries on alpha-stable processes with a special emphasis on bivariate models. Section 3 describes our proposed algorithm for wavelet-domain image fusion, which is based on *fractional lower order moments*. Section 4 compares the performance of our proposed algorithm with the performance of other current wavelet-based fusion techniques applied on two pairs of test images. Finally, in Section 5 we conclude the paper with a short summary.

2 Alpha-Stable Distributions

This section provides a brief, necessary overview of the alpha-stable statistical model used to characterize wavelet coefficients of natural images. Since our interest is in modeling wavelet coefficients, which are symmetric in nature, we restrict our exposition to the case of symmetric alpha-stable distributions. For detailed accounts of the properties of the general stable family, we refer the reader to [9] and [10].

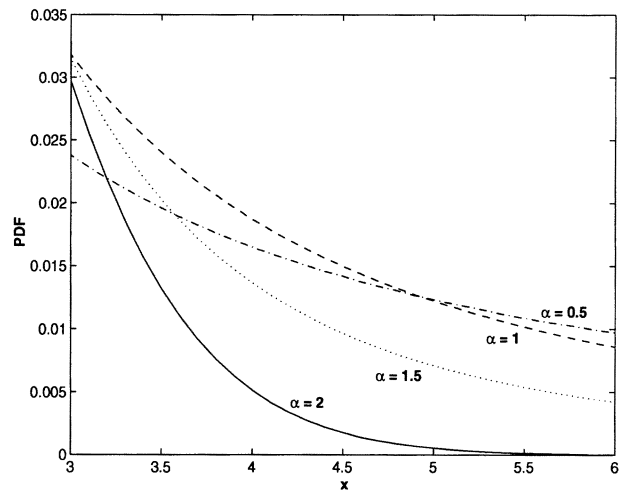


Figure 2: Tail behavior of the $S\alpha S$ probability density functions for $\alpha = 0.5, 1.0$ (Cauchy), 1.5 , and 2.0 (Gaussian). The dispersion parameter is kept constant at $\gamma = 1$.

2.1 Univariate $S\alpha S$ Distributions

The appeal of $S\alpha S$ distributions as a statistical model for signals derives from two main theoretical reasons. First, stable random variables satisfy the stability property which states that linear combinations of jointly stable variables are indeed stable. Second, stable processes arise as limiting processes of sums of independent identically distributed (i.i.d.) random variables via the generalized central limit theorem.

The $S\alpha S$ distribution lacks a compact analytical expression for its probability density function (pdf). Consequently, it is most conveniently represented by its characteristic function

$$\varphi(\omega) = \exp(j\delta\omega - \gamma|\omega|^\alpha) \quad (1)$$

where α is the *characteristic exponent*, taking values $0 < \alpha \leq 2$, δ ($-\infty < \delta < \infty$) is the *location parameter*, and γ ($\gamma > 0$) is the *dispersion* of the distribution. For values of α in the interval $(1, 2]$, the location parameter δ corresponds to the mean of the $S\alpha S$ distribution, while for $0 < \alpha \leq 1$, δ corresponds to its median. The dispersion parameter γ determines the spread of the distribution around its location parameter δ , similar to the variance of the Gaussian distribution.

The characteristic exponent α is the most important parameter of the $S\alpha S$ distribution and it determines the shape of the distribution. The smaller the characteristic exponent α is, the heavier the tails of the $S\alpha S$ density. This implies that random variables following $S\alpha S$ distributions with small characteristic exponents are highly impulsive. One consequence of heavy tails

is that only moments of order less than α exist for the non-Gaussian alpha-stable family members. As a result, stable laws have infinite variance. Gaussian processes are stable processes with $\alpha = 2$ while Cauchy processes result when $\alpha = 1$. Figure 2 shows the tail behavior of several $S\alpha S$ densities including the Cauchy and the Gaussian.

2.2 Bivariate Stable Distributions

Much like univariate stable distributions, bivariate stable distributions are characterized by the stability property and the generalized central limit theorem [10]. The characteristic function of a bivariate stable distribution has the form

$$\varphi(\omega) = \begin{cases} \exp(j\omega^T \delta - \omega^T A \omega) & \text{for } \alpha = 2 \\ \exp(j\omega^T \delta - \int_S |\omega^T s|^\alpha \mu(ds) + j\beta_\alpha(\omega)) & \text{for } 0 < \alpha < 2 \end{cases} \quad (2)$$

where

$$\beta_\alpha(\omega) = \begin{cases} \tan \frac{\alpha\pi}{2} \int_S |\omega^T s|^\alpha \text{sign} |\omega^T s| \mu(ds) & \text{for } \alpha \neq 1, 0 < \alpha < 2 \\ \int_S \omega^T s \log |\omega^T s| \mu(ds) & \text{for } \alpha = 1 \end{cases} \quad (3)$$

and where $\omega = (\omega_1, \omega_2)$, $|\omega| = \sqrt{\omega_1^2 + \omega_2^2}$, and $\delta = (\delta_1, \delta_2)$. S is the unit circle, the measure $\mu(\cdot)$ is called the *spectral measure* of the α -stable random vector and A is a positive semidefinite symmetric matrix.

Unlike univariate stable distribution, bivariate stable distributions form a nonparametric set being thus much more difficult to describe. An exception is the family of multidimensional isotropic stable distributions whose characteristic function has the form

$$\varphi(\omega_1, \omega_2) = \exp(j(\delta_1 \omega_1 + \delta_2 \omega_2) - \gamma |\omega|^\alpha) \quad (4)$$

The distribution is isotropic with respect to the location point (δ_1, δ_2) . The two marginal distributions of the isotropic stable distribution are $S\alpha S$ with parameters $(\delta_1, \gamma, \alpha)$ and $(\delta_2, \gamma, \alpha)$. Since our further developments are in the framework of wavelet analysis, in the following we will assume that $(\delta_1, \delta_2) = (0, 0)$. The bivariate *isotropic Cauchy* and *Gaussian* distributions are special cases for $\alpha = 1$ and $\alpha = 2$, respectively. The bivariate pdf in these two cases can be written as:

$$P_{\alpha, \gamma}(x_1, x_2) = \begin{cases} \frac{\gamma}{2\pi(x_1^2 + x_2^2 + \gamma^2)^{3/2}} & \text{for } \alpha = 1 \\ \frac{1}{4\pi\gamma} \exp[-\frac{x_1^2 + x_2^2}{4\gamma}] & \text{for } \alpha = 2. \end{cases} \quad (5)$$

Figure 3 shows an example of a two-dimensional Cauchy surface. As in the case of the univariate $S\alpha S$ density function, when $\alpha \neq 1$ and $\alpha \neq 2$, no closed form expressions exist for the density function of the bivariate stable random variable, but a numerical calculation algorithm is available [11].

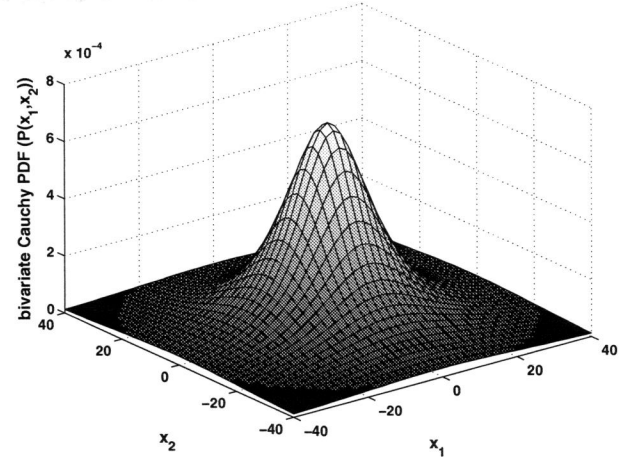


Figure 3: Two dimensional Cauchy density surface ($\alpha = 1, \gamma = 15$).

3 FLOM based match and salience measures

Following the arguments in [2], we settle by considering two different modes for fusion: selection and averaging. The overall fusion rule is determined by two measures: a match measure that determines which of the two modes is to be employed and a salience measure that determines which wavelet coefficient in the pair will be copied in the fused subband. Unlike the approach in [2], our match and salience measures are not based on classical second order moments and correlation. Instead, we introduce new measures, which can be defined based on the so-called *fractional lower order moments* (FLOM's) of $S\alpha S$ distributions. Specifically, we employ the dispersion γ , estimated in a neighborhood around the reference wavelet coefficients as a salience measure, while the *symmetric coefficient of covariation* is introduced in order to account for the similarities between corresponding patterns in the two subbands to be fused.

3.1 Local dispersion estimation

The FLOM's of a $S\alpha S$ random variable with zero location parameter and dispersion γ are given by [10]:

$$E|X|^p = C(p, \alpha) \gamma^{\frac{p}{\alpha}} \quad \text{for } -1 < p < \alpha \quad (6)$$

where

$$C(p, \alpha) = \frac{2^{p+1} \Gamma(\frac{p+1}{2}) \Gamma(-\frac{p}{\alpha})}{\alpha \sqrt{\pi} \Gamma(-\frac{p}{2})} \quad (7)$$

and $\Gamma(\cdot)$ is the Gamma function. Using the above expression for the p^{th} order moment of X , one can show

that [12]

$$E(|X|^p)E(|X|^{-p}) = \frac{2 \tan(p\pi/2)}{\alpha \sin(p\pi/\alpha)} \quad (8)$$

where $0 < p < \min(\alpha, 1)$ and therefore

$$\text{sinc}\left(\frac{p\pi}{\alpha}\right) = \frac{2 \tan(p\pi/2)}{p\pi E(|X|^p)E(|X|^{-p})} \quad (9)$$

The estimation process involves inversion of the sinc function in (9) to obtain α and then γ can be found using

$$\gamma = \left(\frac{E(|X|^p)}{C(p, \alpha)}\right)^{\alpha/p} \quad (10)$$

In order to avoid having to invert a sinc function as well as choosing the moment exponent p in the above estimation process, one can rewrite $E(|X|^p)$ as $E(e^{p \log |X|})$ and define a new random variable $Y = \log |X|$. Hence, it can be shown [12] that the mean and variance of Y are respectively given by

$$E(Y) = \frac{\alpha - 1}{\alpha} C_e + \frac{\log \gamma}{\alpha} \quad (11)$$

where $C_e = 0.5772166 \dots$ is the Euler constant, and

$$E([Y - E(Y)]^2) = \frac{\pi^2}{12} \frac{\alpha^2 + 2}{\alpha^2} \quad (12)$$

The estimation process simply involves now solving (12) for α and substituting back in (11) to find the value of the dispersion parameter γ .

3.2 Symmetric covariation coefficient

The notion of covariance between two random variables plays an important role in the second-order moment theory. However, due to the lack of finite variance, covariances do not exist either on the space of SaS random variables. Instead, quantities like covariations or codifferences, which under certain circumstances play analogous roles for SaS random variables to the one played by covariance for Gaussian random variables have been introduced [9]. Specifically, let X_1 and X_2 be jointly SaS random variables with $\alpha > 1$, zero location parameters and dispersions γ_{X_1} and γ_{X_2} respectively. The covariation of X_1 with X_2 is defined in terms of the previously introduced FLOM's by [9]

$$[X_1, X_2]_\alpha = \frac{E(X_1 X_2^{<p-1>})}{E(|X_2|^p)} \gamma_{X_2} \quad (13)$$

where $x^p = |x|^p \text{sign}(x)$. Moreover, the covariation coefficient of X_1 with X_2 , is the quantity

$$\lambda_{X_1, X_2} = \frac{[X_1, X_2]_\alpha}{[X_2, X_2]_\alpha} \quad \text{for any } 1 \leq p < \alpha \quad (14)$$

Unfortunately, it is a well-known fact that the covariation coefficient in (14) is neither symmetric nor bounded [9]. Therefore, we propose the use of a symmetrized and normalised version of the above quantity, which enable us to define a new match measure for SaS random vectors. The symmetric covariation coefficient can be simply defined as

$$\text{Corr}_\alpha(X_1, X_2) = \lambda_{X_1, X_2} \lambda_{X_2, X_1} = \frac{[X_1, X_2]_\alpha [X_2, X_1]_\alpha}{[X_1, X_1]_\alpha [X_2, X_2]_\alpha} \quad (15)$$

Garel et. al [13] have shown that the symmetric covariation coefficient is bounded, taking values between -1 and 1. In our implementation, the above similarity measure is computed in a square-shaped neighborhood of size 7×7 around each reference coefficient.

3.3 Summary of fusion algorithm

- I) Decompose each input image into subbands.
- II) For each subband pair (except the lowpass residuals):

1. Estimate neighborhood dispersions, γ_{X_1} and γ_{X_2} using (12) and (11)
2. Compute neighborhood symmetric covariation coefficient $\text{Corr}_\alpha(X_1, X_2)$ using (15)
3. Calculate the fused coefficients using the formula $Y = w_1 X_1 + w_2 X_2$ as follows
 - if $\text{Corr}_\alpha \leq T$ then $w_{\min} = 0$ & $w_{\max} = 1$
 - elseif $\text{Corr}_\alpha > T$ then $w_{\min} = 0.5 - 0.5 \frac{1 - \text{Corr}_\alpha}{1 - T}$ & $w_{\max} = 1 - w_{\min}$
 - if $\gamma_{X_1} > \gamma_{X_2}$ then $w_1 = w_{\max}$ and $w_2 = w_{\min}$
 - else $w_1 = w_{\min}$ and $w_2 = w_{\max}$

- III) Average coefficients in lowpass residual

- IV) Reconstruct the fused image from the processed subbands and the lowpass residual

4 Results

Appropriate methodology for comparing different image fusion methods generally depends on the application. For example, in applications like medical image fusion, the ultimate goal is to combine perceptually salient image elements such as edges and high contrast regions. Evaluation of fusion techniques in such situation can only be effective based on visual assessment. There are also applications (e.g. multifocus image fusion) when computational measures could be employed. Several qualitative measures have been

proposed for this purpose. For example, in [14] Zhang and Blum have used a mutual information criterion, the root mean square error as well as a measure representing the percentage of correct decisions. Unfortunately, all these measures involve the existence of a reference image for their computation, which in practice is generally not available. Moreover, the problem with the mutual information measure, or with any other metric, is their connection to the visual interpretation of an human observer. On analyzing an image, a human observer does not compute any such measure. Hence, in order to study the merit of the proposed fusion algorithm, we chose different images, applied the algorithm, and visually evaluated the fused image. We were interested in performing experiments on images of different types and with various content in order to be able to obtain results, which we could claim to be general enough. Thus, the first example shows the “Cathe” image pair that can be found in Matlab’s Image Processing Toolbox. As a second example, we chose to illustrate the fusion of two medical images, a magnetic resonance (MR) image and a computer tomography (CT). The results of these experiments are shown in Fig. 4 and Fig. 5.

The figures show results obtained using four different methods, including a maximum selection (MS) scheme, a pixel averaging (PA) scheme, the weighted average (WA) method [2], and our proposed algorithm. Clearly, the last two techniques outperform the MS and PA methods. Although further qualitative evaluation in this way is highly subjective, it seems that the best results are achieved by our proposed technique. It appears that our system performs like a feature detector, retaining the features that are clearly distinguishable in each of the two input images.

5 Summary

In this work, we proposed a novel image fusion algorithm for the case when the data to be fused exhibit heavy tails with no convergent second- or higher-order moments. Specifically, our developments were based on recently published results showing that wavelet decomposition coefficients of images are best modeled by $S\alpha S$ distributions. Due to the lack of second or higher order moments for the general alpha-stable family members, we proposed new salience and match measures, which are based on the so-called *fractional lower order moments*. Thus, in the multiscale domain, we employed the local dispersion of wavelet coefficients as a salience measure, while *symmetric covariation coefficients* were computed in order to account for the similarities between corresponding patterns in the two subbands to be fused. Simulation results showed that our method achieves better performance in comparison

with previously proposed pixel-level fusion approaches.

References

- [1] J. J. Lewis, R. J. O’Callaghan, S. G. Nikolov, D. Bull, and N. Canagarajah, “Region-based image fusion using complex wavelets,” *Proc. Seventh Int’l Conf. Inform. Fus.*, pp. 555–562, June 2004.
- [2] P. J. Burt and R. J. Kolczynski, “Enhanced image capture through fusion,” *Proc. Fourth Int’l Conf. Comp. Vis.*, pp. 173–182, 1993.
- [3] O. Rockinger, “Image sequence fusion using a shift-invariant wavelet transform,” *Proc. IEEE Int’l Conf. on Image Proc.*, pp. 288–291, Oct. 1997.
- [4] P. Hill, N. Canagarajah, and D. Bull, “Image fusion using complex wavelets,” *Proc. 13th British Machine Vis. Conf. (BMVC-2002)*, pp. 487–496, Sep. 2002.
- [5] A. Achim, A. Bezerianos, and P. Tsakalides, “Novel Bayesian multiscale method for speckle removal in medical ultrasound images,” *IEEE Trans. Med. Imag.*, vol. 20, pp. 772–783, Aug. 2001.
- [6] A. Achim, P. Tsakalides, and A. Bezerianos, “SAR image denoising via Bayesian wavelet shrinkage based on heavy-tailed modeling,” *IEEE Trans. Geosci. and Remote Sensing*, vol. 41, pp. 1773–1784, Aug. 2003.
- [7] N. G. Kingsbury, “Image processing with complex wavelets,” *Phil. Trans. R. Soc. London A*, vol. 357, pp. 2543–2560, Sept. 1999.
- [8] N. G. Kingsbury, “Complex wavelets for shift invariant analysis and filtering,” *Appl. Comput. Harmon. Anal.*, vol. 10, pp. 234–253, May 2001.
- [9] G. Samorodnitsky and M. S. Taqqu, *Stable Non-Gaussian Random Processes: Stochastic Models with Infinite Variance*. New York: Chapman and Hall, 1994.
- [10] C. L. Nikias and M. Shao, *Signal Processing with Alpha-Stable Distributions and Applications*. New York: John Wiley and Sons, 1995.

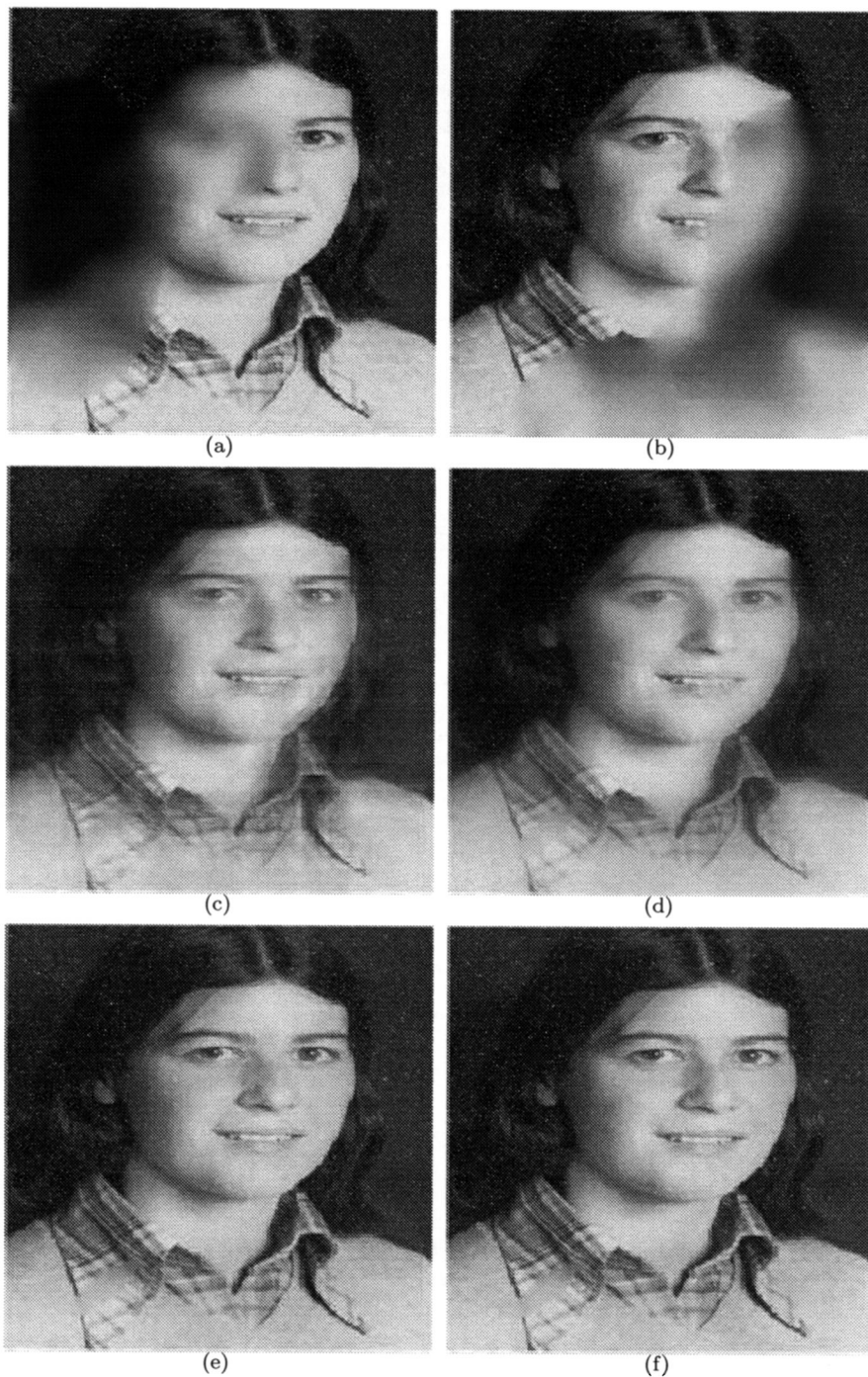


Figure 4: Results of various fusion methods. (a) First test image. (b) Second test image. (c) Image fused using MS method. (d) Image fused using PA method. (e) Image fused using WA method. (f) Image fused using our proposed method.

- [11] J. P. Nolan, "Multivariate stable distributions: approximation, estimation, simulation and identification," in *A Practical Guide to Heavy Tails* (R. J. Adler, R. E. Feldman, and M. S. Taquu, eds.), Boston: Birkhauser, 1998.
- [12] X. Ma and C. L. Nikias, "Parameter estimation and blind channel identification in impulsive signal environment," *IEE Tran. Sign. Proc.*, vol. 43, pp. 2884–2897, Dec. 1995.
- [13] B. Garel, L. d'Estampes, and D. Tjostheim, "Re-

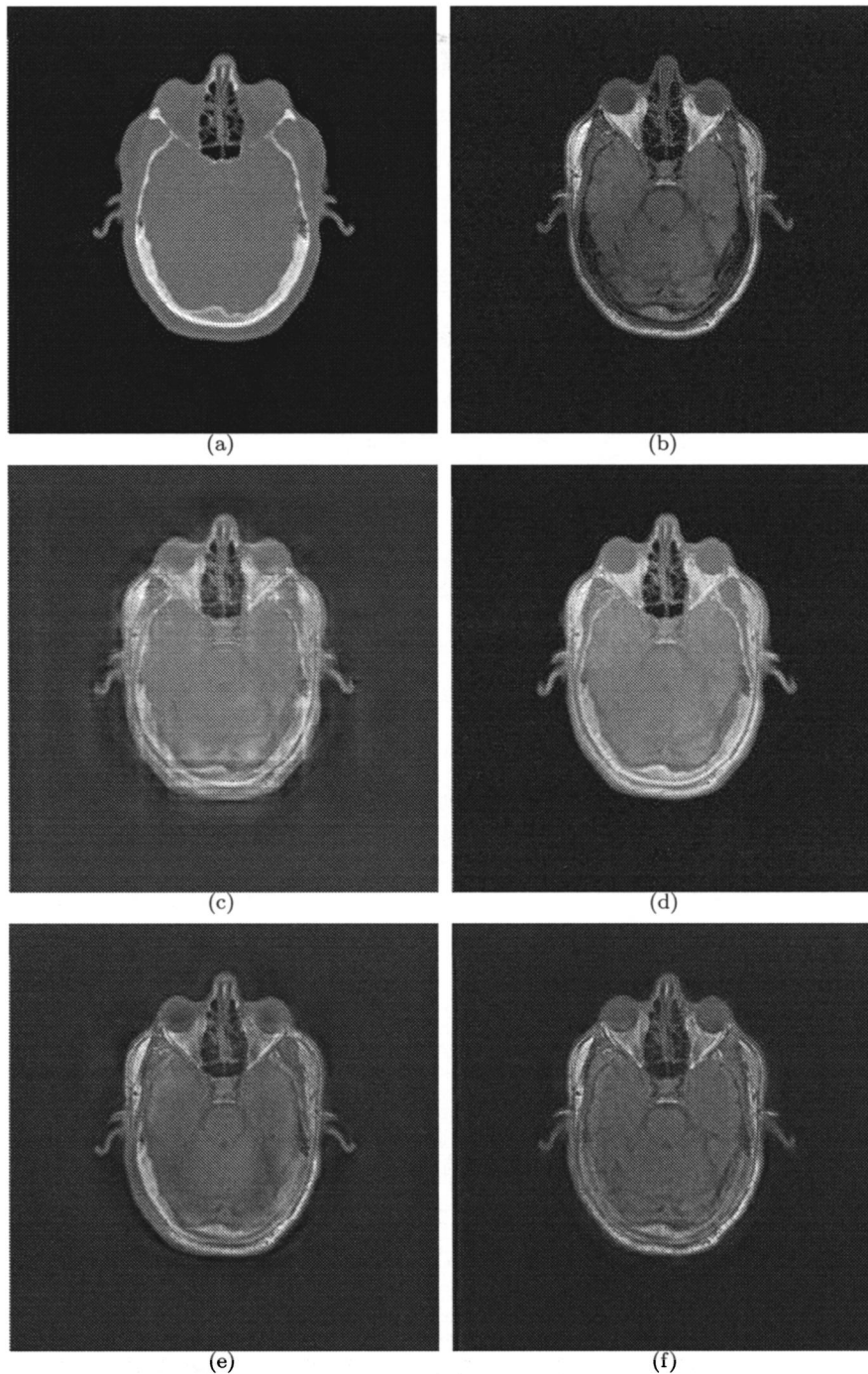


Figure 5: Results of various fusion methods. (a) MR test image. (b) CT test image. (c) Image fused using MS method. (d) Image fused using PA method. (e) Image fused using WA method. (f) Image fused using our proposed method.

vealing some unexpected dependence properties of linear combinations of stable random variables using symmetric covariation,” *Comm. Stat. - Theory and Methods*, vol. 33, no. 4, pp. 769–786, 2004.

[14] Z. Zhang and R. S. Blum, “A categorization

of multiscale-decomposition-based image fusion schemes with a performance study for a digital camera application,” *Proc. IEEE*, pp. 1315–1326, Aug. 1999.